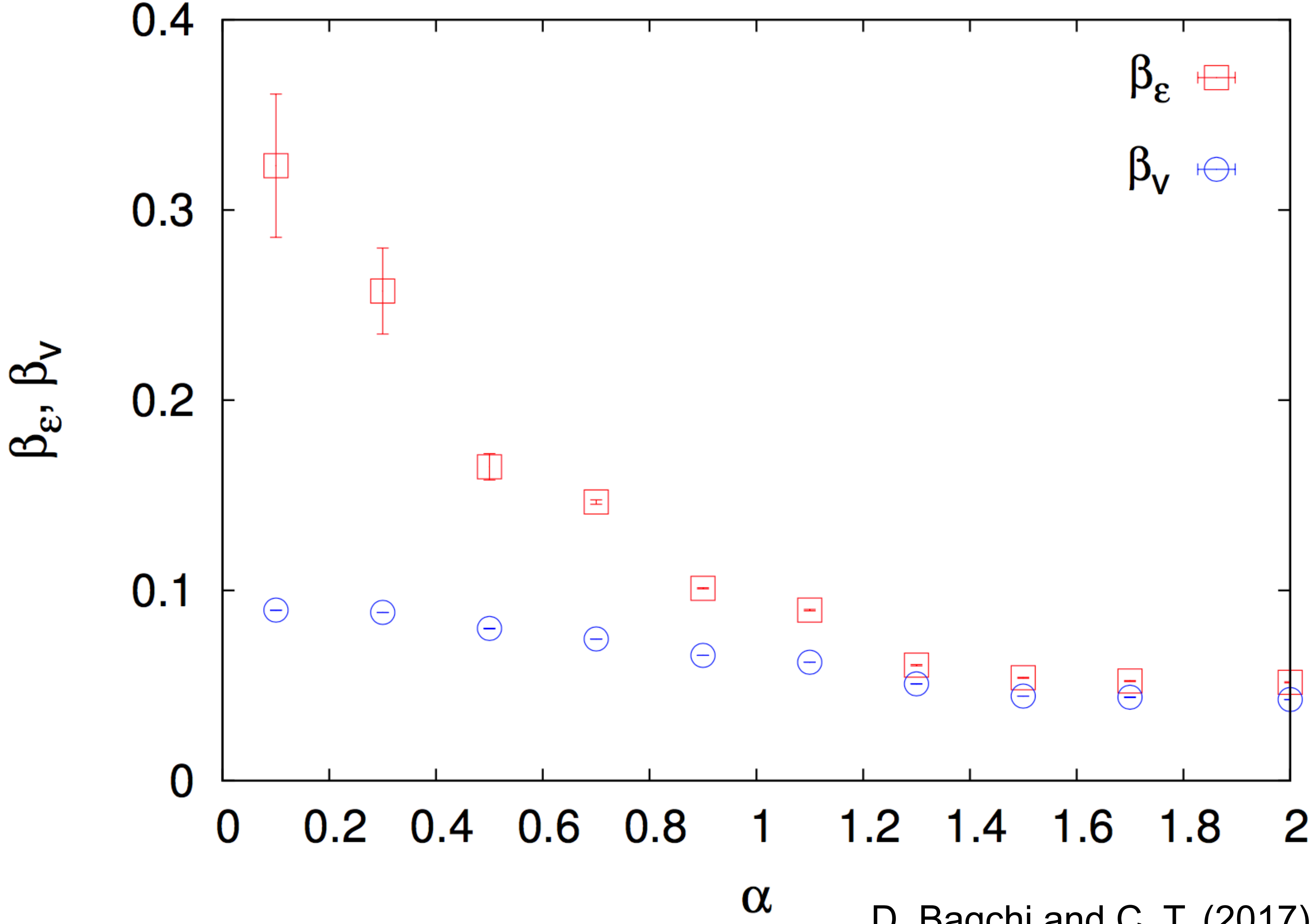

D. Bagchi and C. T. (2017)

BOLTZMANN-GIBBS STATISTICAL MECHANICS

(Maxwell 1860, Boltzmann 1872, Gibbs ≤ 1902)

Entropy $S_{BG} = -k \sum_{i=1}^{W} p_i \ln p_i$

Internal energy $U_{BG} = \sum_{i=1}^{W} p_i E_i$

Equilibrium distribution $p_i = e^{-\beta E_i} / Z_{BG} \quad \left(Z_{BG} \equiv \sum_{j=1}^{W} e^{-\beta E_j} \right)$

Paradigmatic differential equation $\left. \begin{array}{l} \frac{dy}{dx} = ay \\ y(0) = 1 \end{array} \right\} \Rightarrow y = e^{ax}$

	x	a	$y(x)$
Equilibrium distribution	E_i	$-\beta$	$Z\, p(E_i)$
Sensitivity to initial conditions	t	λ	$\xi \equiv \lim_{\Delta x(0)\to 0} \frac{\Delta x(t)}{\Delta x(0)} = e^{\lambda t}$
Typical relaxation of observable O	t	$-1/\tau$	$\Omega \equiv \frac{O(t)-O(\infty)}{O(0)-O(\infty)} = e^{-t/\tau}$

S_{BG} → additive, concave, Lesche-stable, finite entropy production

NONEXTENSIVE STATISTICAL MECHANICS

(C. T. 1988, E.M.F. Curado and C. T. 1991, C. T., R.S. Mendes and A.R. Plastino 1998)

Entropy
$$S_q = k\left(1-\sum_{i=1}^{W} p_i^q\right)/(q-1)$$

Internal energy
$$U_q = \sum_{i=1}^{W} p_i^q E_i \Big/ \sum_{j=1}^{W} p_j^q$$

Stationary state distribution
$$p_i = e_q^{-\beta_q (E_i - U_q)} / Z_q \qquad \left(Z_q \equiv \sum_{j=1}^{W} e_q^{-\beta_q (E_j - U_q)} \right)$$

Paradigmatic differential equation
$$\left.\begin{array}{l} \dfrac{dy}{dx} = a\, y^{q} \\ y(0) = 1 \end{array}\right\} \Rightarrow \qquad y = e_q^{ax} \equiv [1 + (1-q)ax]^{\frac{1}{1-q}}$$

	x	a	$y(x)$
Stationary state distribution	E_i	$-\beta_{q_{stat}}$	$Z_{q_{stat}}\, p(E_i)$ (typically $q_{stat} \geq 1$)
Sensitivity to initial conditions	t	$\lambda_{q_{sen}}$	$\xi = e_{q_{sen}}^{\lambda_{q_{sen}} t}$ (typically $q_{sen} \leq 1$)
Typical relaxation of observable O	t	$-1/\tau_{q_{rel}}$	$\Omega = e_{q_{rel}}^{-t/\tau_{q_{rel}}}$ (typically $q_{rel} \geq 1$)

S_q → nonadditive, concave, Lesche-stable, finite entropy production

C. T., Physica A **340**,1 (2004)

Prediction of the q - triplet: C. T., Physica A **340**,1 (2004)

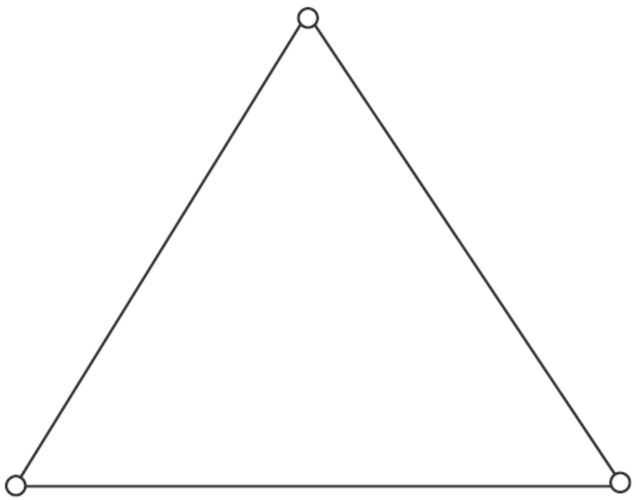

Fig. 2. The triangle of the basic values of q, namely those associated with sensitivity to the initial conditions, relaxation and stationary state. For the most relevant situations we expect $q_{sen} \leqslant 1$, $q_{rel} \geqslant 1$ and $q_{stat} \geqslant 1$. These indices are presumably inter-related since they all descend from the particular dynamical exploration that the system does of its full phase space. For example, for long-range Hamiltonian systems characterized by the decay exponent α and the dimension d, it could be that q_{stat} decreases from a value above unity (e.g., 2 or $\frac{3}{2}$) to unity when α/d increases from zero to unity. For such systems one expects relations like the (particularly simple) $q_{stat} = q_{rel} = 2 - q_{sen}$ or similar ones. In any case, it is clear that, for $\alpha/d > 1$ (i.e., when BG statistics is known to be the correct one), one has $q_{stat} = q_{rel} = q_{sen} = 1$. All the weakly chaotic systems focused on here are expected to have well defined values for q_{sen} and q_{rel}, but only those associated with a Hamiltonian are expected to *also* have a well defined value for q_{stat}.

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Physica A 356 (2005) 375–384

PHYSICA A

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Triangle for the entropic index q of non-extensive statistical mechanics observed by Voyager 1 in the distant heliosphere

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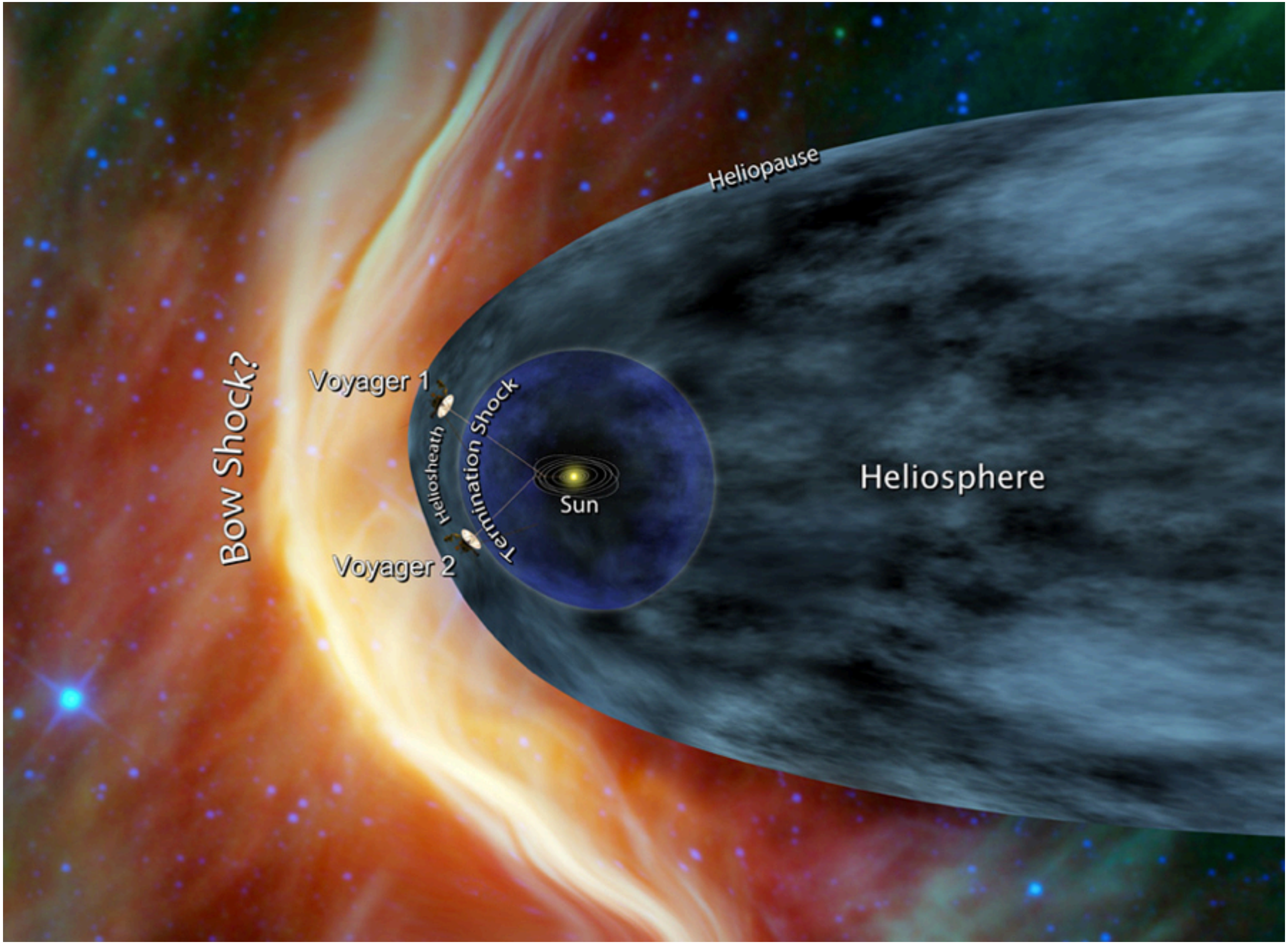
Laboratory for Solar and Space Physics, Code 612.2, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA

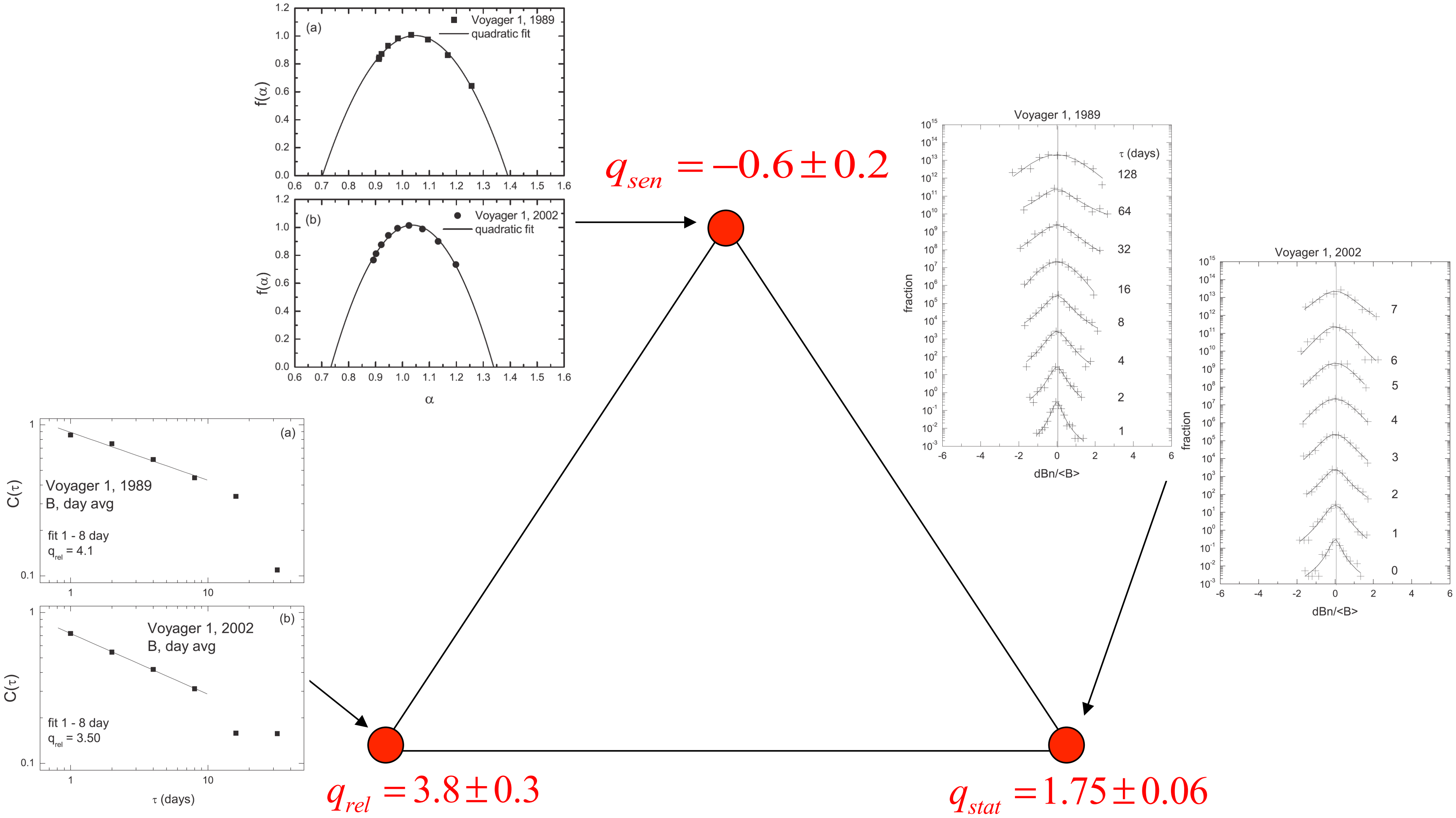
Received 10 June 2005
Available online 11 July 2005


Heliopause
Bow Shock?
Voyager 1
Heliosheath
Termination Shock
Sun
Voyager 2
Heliosphere

SOLAR WIND: Magnetic Field Strength

L.F. Burlaga and A. F.-Vinas (2005) / NASA Goddard Space Flight Center; Physica A **356**, 375 (2005)

[Data: Voyager 1 spacecraft (1989 and 2002); 40 and 85 AU; daily averages]

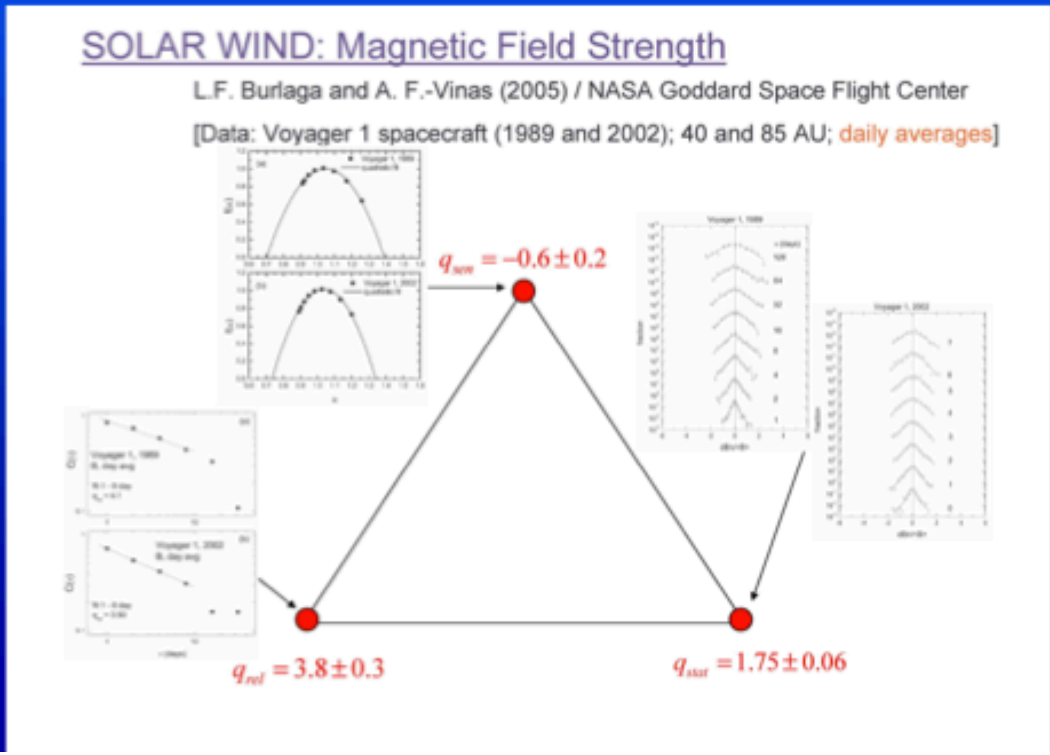


$$q_{sens} < 1 < q_{stat} < q_{rel}$$

Office for Outer Space Affairs
United Nations Office at Vienna

IHY 2007: VOYAGER 1: Fundamental Physics

The atmosphere of the Sun beyond a few solar radii, known as HELIOSPHERE, is fully ionized plasma expanding at supersonic speeds, carrying solar magnetic fields with it. This solar wind is a driven non-linear non-equilibrium system. The Sun injects matter, momentum, energy, and magnetic fields into the heliosphere in a highly variable way. Voyager 1 observed magnetic field strength variations in the solar wind near 40 AU during 1989 and near 85 AU during 2002. Tsallis' non-extensive statistical mechanics, a generalization of Boltzmann-Gibbs statistical mechanics, allows a physical explanation of these magnetic field strength variations in terms of departure from thermodynamic equilibrium in an unique way:



Asymptotically scale-invariant occupancy of phase space makes the entropy S_q extensive

Constantino Tsallis*†‡, Murray Gell-Mann*‡, and Yuzuru Sato*

*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501; and †Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil

Contributed by Murray Gell-Mann, July 25, 2005

TGS algebra: C. T., M. Gell-Mann and Y. Sato, PNAS **102** (2005) 15377

$\mu: \;\; q \rightarrow 2\text{-}q \quad$ (*additive duality*)

$\nu: \;\; q \rightarrow 1/q \quad$ (*multiplicative duality*)

hence $\mu^2 = \nu^2 = 1$ (self-duality), $q=1$ is a fixed point,

and $\mu\nu\mu\nu$... generates an algebra

$$\left[\mu\nu = (\nu\mu)^{-1} \rightarrow \frac{1}{2-q}; \;\; \nu\mu \rightarrow 2-\frac{1}{q}; \; ... \right]$$

Playing with additive duality $(q \to 2-q)$

and with multiplicative duality $(q \to 1/q)$

(*and using numerical results related to the* $q-$*generalized central limit theorem*)

we conjecture

$$q_{rel} + \frac{1}{q_{sen}} = 2 \qquad \text{and} \qquad q_{stat} + \frac{1}{q_{rel}} = 2$$

hence $$1 - q_{sen} = \frac{1 - q_{stat}}{3 - 2\, q_{stat}}$$

hence only one independent!

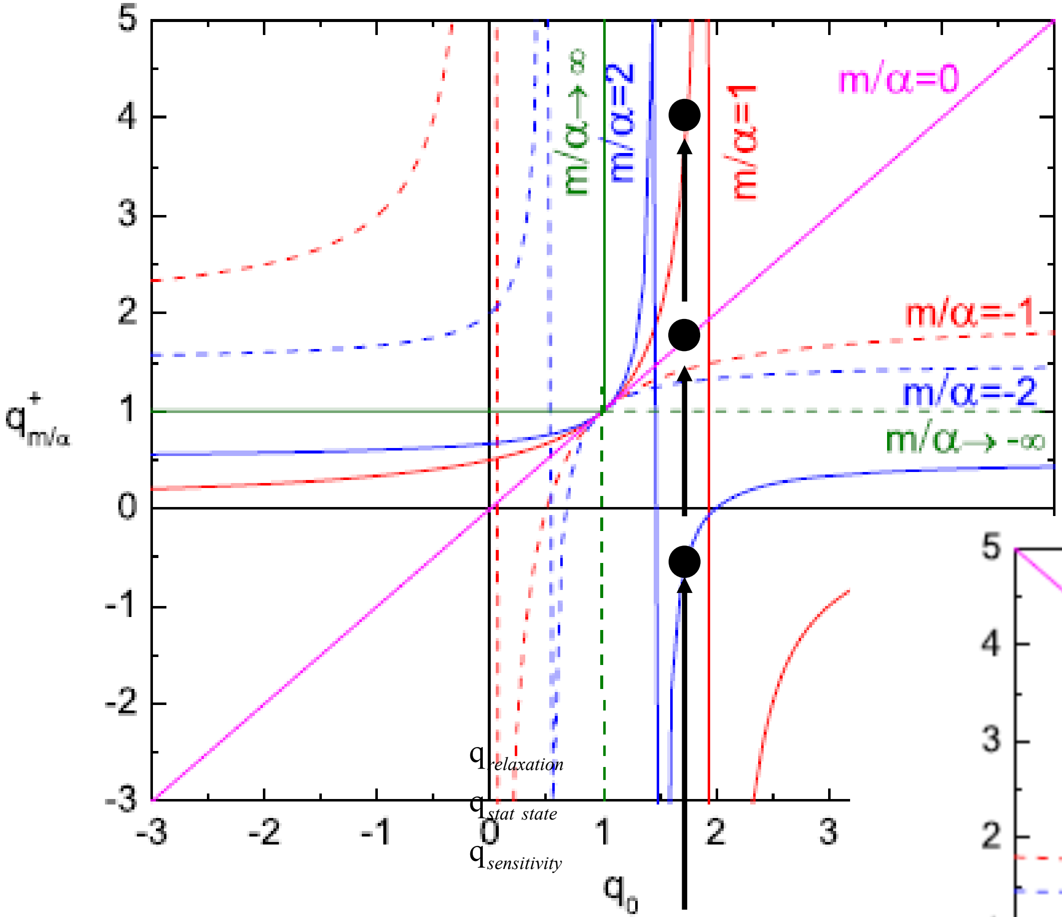
Burlaga and Vinas (NASA) most precise value of the $q-$*triplet is*

$$q_{stat} = 1.75 = 7/4$$

hence $q_{sen} = -0.5 = -1/2$ (*consistent with* $q_{sen} = -0.6 \pm 0.2$!)

and $q_{rel} = 4$ (*consistent with* $q_{rel} = 3.8 \pm 0.3$!)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)



$$\frac{1}{1-q^{+}_{m/\alpha}} = \frac{1}{1-q_0} + \frac{m}{\alpha}$$

$$\frac{1}{1-q^{-}_{m/\alpha}} = \frac{1}{q_0-1} + \frac{m}{\alpha}$$

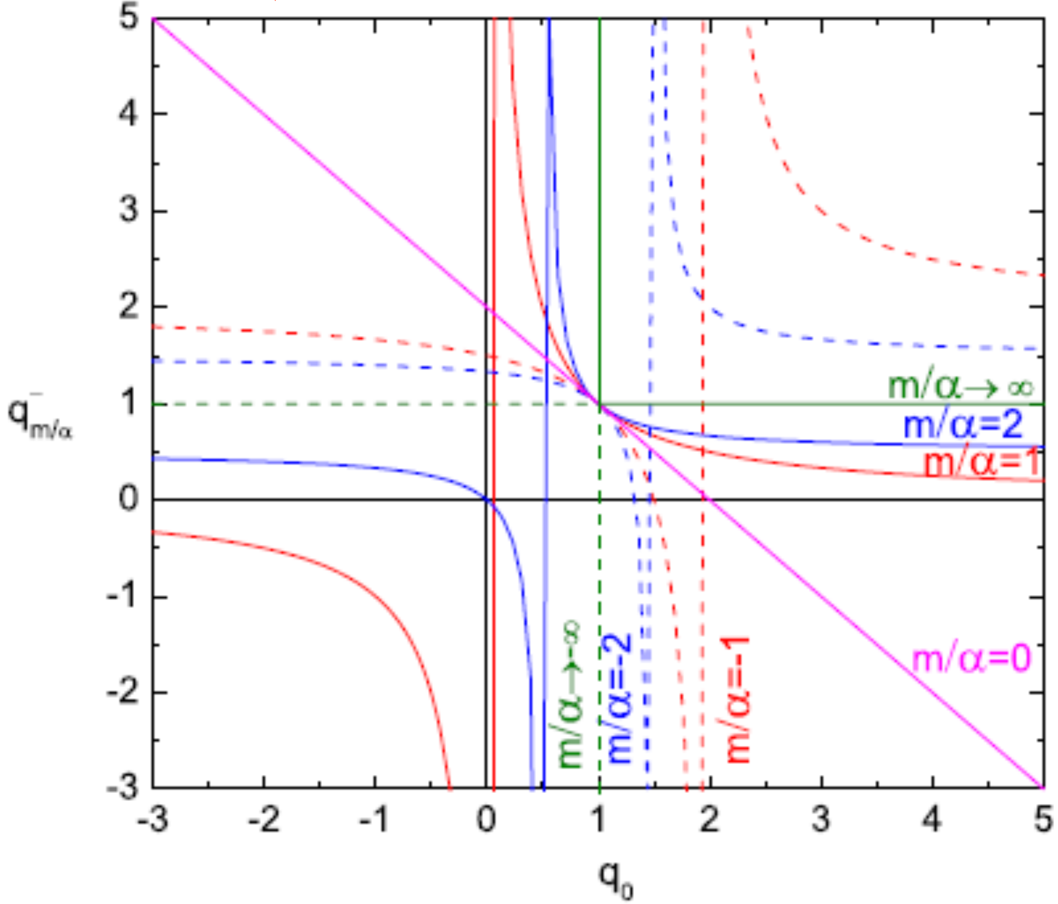
$$(0 < \alpha \le 2;\ m = 0, \pm 1, \pm 2, \ldots)$$

solar wind q-triplet

$q_{relaxation}$

$q_{stat\ state}$

$q_{sensitivity}$



C. T., Braz J Phys **39**, 337 (2009)

$$\varepsilon_{sen} \equiv 1 - q_{sen} = 1 - (-1/2) = 3/2$$

$$\varepsilon_{rel} \equiv 1 - q_{rel} = 1 - 4 = -3$$

$$\varepsilon_{stat} \equiv 1 - q_{stat} = 1 - 7/4 = -3/4$$

We verify

$$\varepsilon_{stat} = \frac{\varepsilon_{sen} + \varepsilon_{rel}}{2} \qquad \text{(arithmetic mean!)}$$

$$\varepsilon_{sen} = \sqrt{\varepsilon_{stat} \ \varepsilon_{rel}} \qquad \text{(geometric mean!)}$$

$$\varepsilon_{rel}^{-1} = \frac{\varepsilon_{sen}^{-1} + \varepsilon_{stat}^{-1}}{2} \qquad \text{(harmonic mean!)}$$

N.O. Baella (2008)

Eur. Phys. J. Special Topics 226, 455–466 (2017)
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SPECIAL TOPICS

Regular Article

Generalization of the possible algebraic basis of *q*-triplets

Constantino Tsallis[1,2,a]

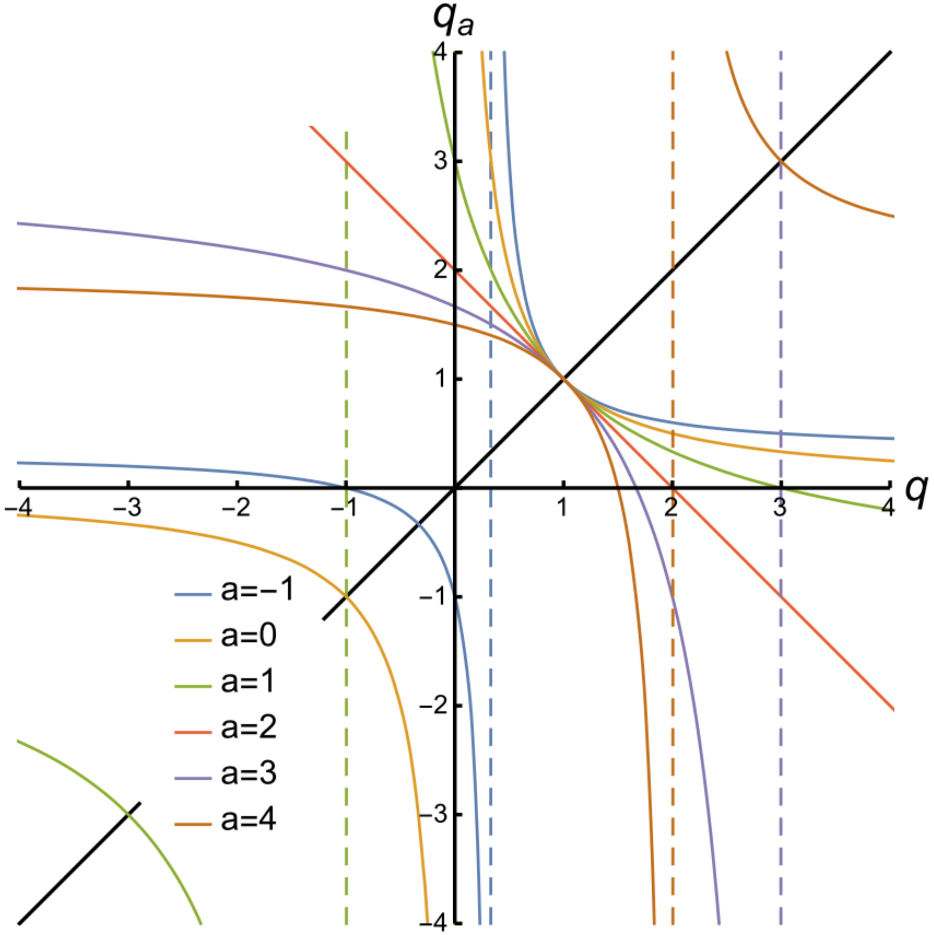
$$q \to \frac{(a+2)-aq}{a-(a-2)q} \equiv f_a(q),$$

equivalently $$\frac{1}{1-q} \to \frac{1}{q-1} - \frac{a}{2} + 1$$

$$(a \in R)$$

$$f_2(q) = 2-q$$

$$f_0(q) = 1/q$$



Eur. Phys. J. Special Topics **226**, 455–466 (2017)
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DOI: 10.1140/epjst/e2016-60159-x

THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

Regular Article

Generalization of the possible algebraic basis of *q*-triplets

Constantino Tsallis[1,2,a]

$$\mu: \; q \rightarrow \frac{(a+2)-aq}{a-(a-2)q} \equiv f_a(q), \quad \text{equivalently} \quad \frac{1}{1-q} \rightarrow \frac{1}{q-1} - \frac{a}{2} + 1 \quad (a \in R)$$

$$\nu: \; q \rightarrow \frac{(b+2)-bq}{b-(b-2)q} \equiv f_b(q), \quad \text{equivalently} \quad \frac{1}{1-q} \rightarrow \frac{1}{q-1} - \frac{b}{2} + 1 \quad (b \in R)$$

hence $\mu^2 = \nu^2 = 1$ (self-duality), $q = 1$ is a fixed point,

$f_2(q) = 2 - q$

$f_0(q) = 1/q$

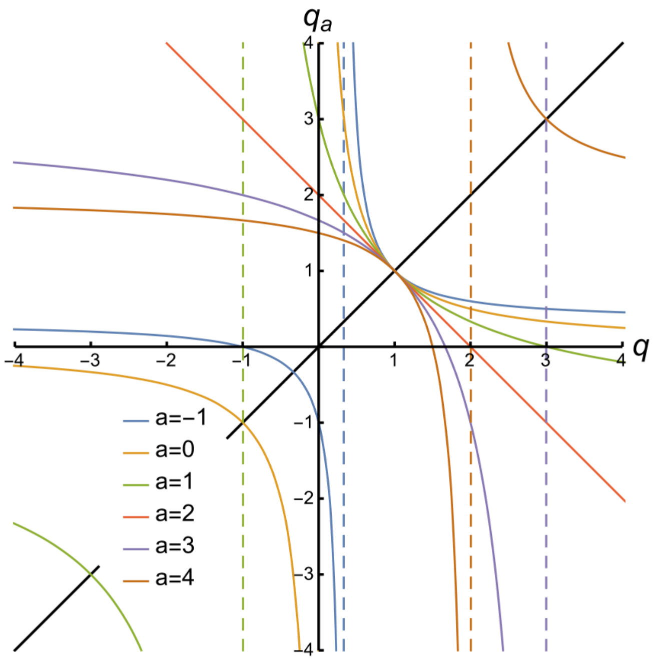
and $\mu\nu\mu\nu...$ generalizes the TGS algebra $[\mu\nu = (\nu\mu)^{-1} \rightarrow f_a(f_b(q));\; ...]$

$$\mu\nu: \quad \frac{1}{1-q} \to \frac{1}{1-q} + \frac{b-a}{2}$$

$$\nu\mu = (\mu\nu)^{-1}: \quad \frac{1}{1-q} \to \frac{1}{1-q} + \frac{a-b}{2}$$

$$(\mu\nu)^{z}: \quad \frac{1}{1-q} \to \frac{1}{1-q} + z\,\frac{b-a}{2}$$

$$(z = 0,\ \pm 1,\ \pm 2, \ldots)$$



or equivalently

$$\frac{2}{(b-a)(1-q)} \to \frac{2}{(b-a)(1-q)} + z$$

$$(z = 0,\ \pm 1,\ \pm 2, \ldots)$$

FURTHER GENERALIZATION

C. T. (Springer, 2017), in press

Statistical mechanics for complex systems: On the structure of q-triplets*

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The most general self-dual ratio of linear functions of q, which has the $q = 1$ fixed point, is given by

$$q \to \frac{a_1 - a_2 q}{a_2 - (2a_2 - a_1)q} \equiv f_{a_1, a_2}(q) \quad (a_1 \in R,\ a_2 \in R)$$

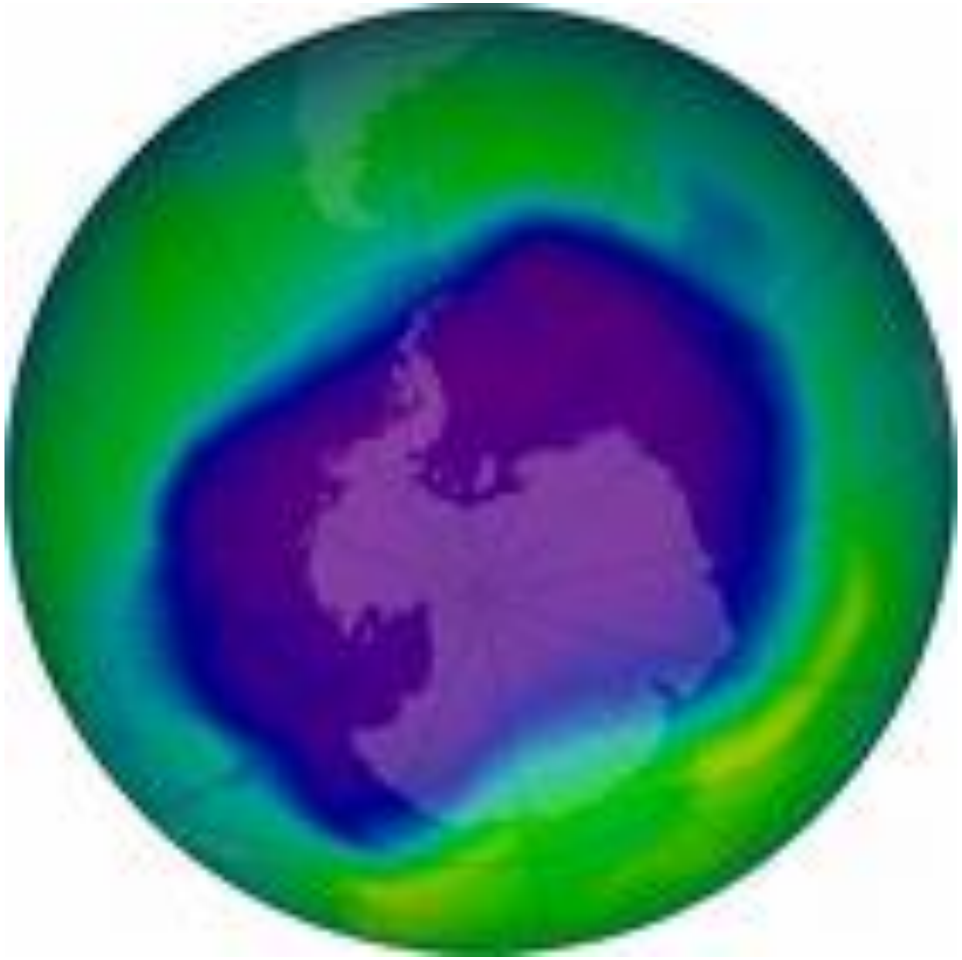
Particular case:

$$a_1 = a + 2$$

$$a_2 = a$$

recovers the previous transformation.

OZONE LAYER HOLE



10-50 Km above Earth

It absorbs 93-99% of the sun´s high frequency ultraviolet light

Physica A 389 (2010) 1829–1833

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Tsallis' q-triplet and the ozone layer

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A R T I C L E   I N F O

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A B S T R A C T

Tsallis' q-triplet [C. Tsallis, Dynamical scenario for nonextensive statistical mechanics, Physica A 340 (2004) 1–10] is the best empirical quantifier of nonextensivity. Here we study it with reference to an experimental time-series related to the daily depth-values of the stratospheric ozone layer. Pertinent data are expressed in Dobson units and range from 1978 to 2005. After the evaluation of the three associated Tsallis' indices one concludes that nonextensivity is clearly a characteristic of the system under scrutiny.

Original data = mean value + long range tendency + annual oscillation + quasi-biannual one + Z_n

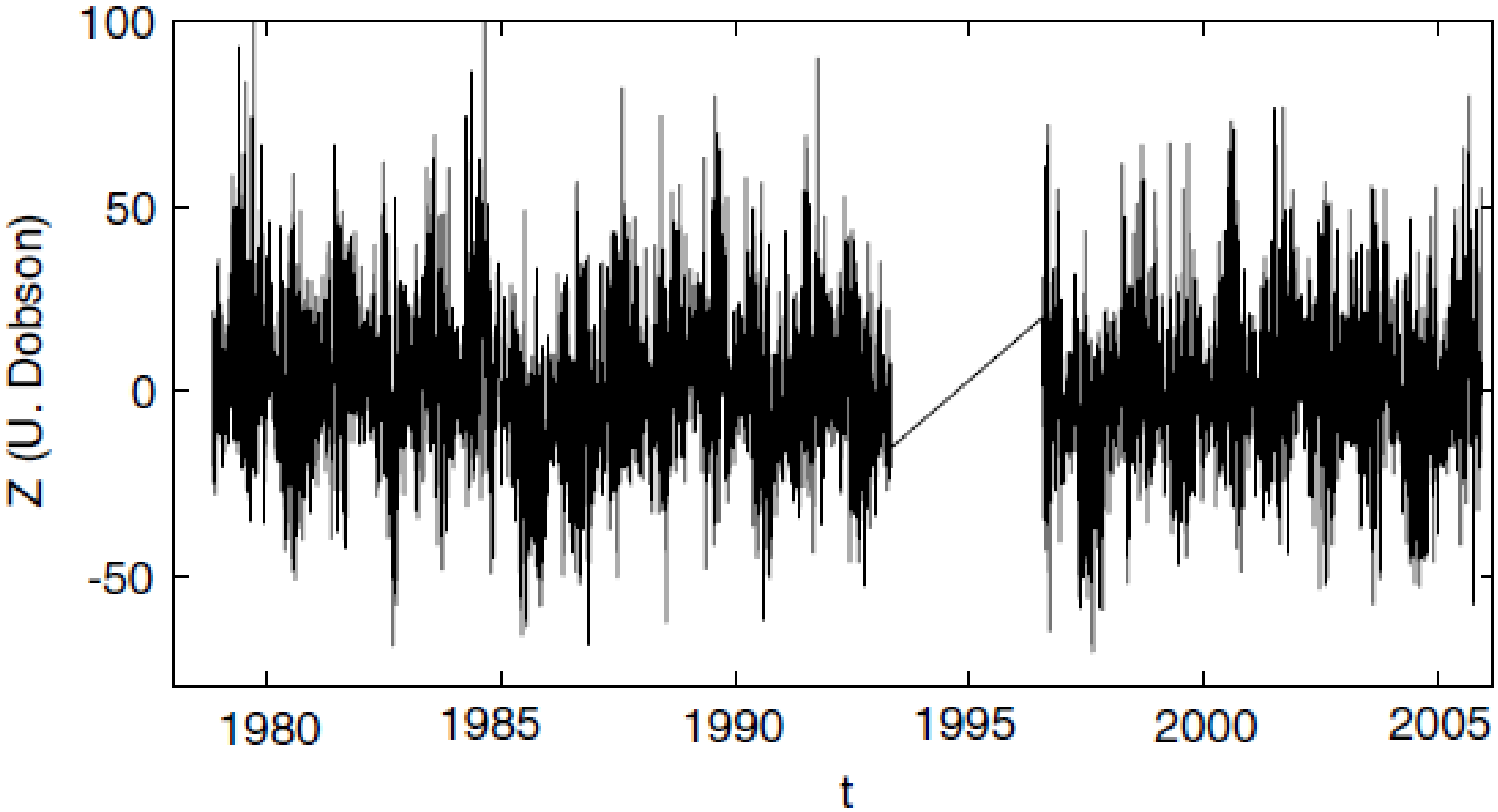

Fig. 1. Time-series Z_n. Daily values of the ozone layer over Buenos Aires city.

G.L. Ferri, M.F.R. Savio and A. Plastino, Physica A **389**, 1829 (2010)

$$q_{stat} = \ 1.32 \pm 0.06$$

$$q_{sens} = -8.1 \pm 0.02$$

$$q_{rel} = \ 1.89 \pm 0.02$$

hence

$$q_{sens} < 1 < q_{stat} < q_{rel}$$

G.L. Ferri, M.F.R. Savio and A. Plastino, Physica A **389**, 1829 (2010)

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Tsallis' statistics in the variability of El Niño/Southern Oscillation during the Holocene epoch

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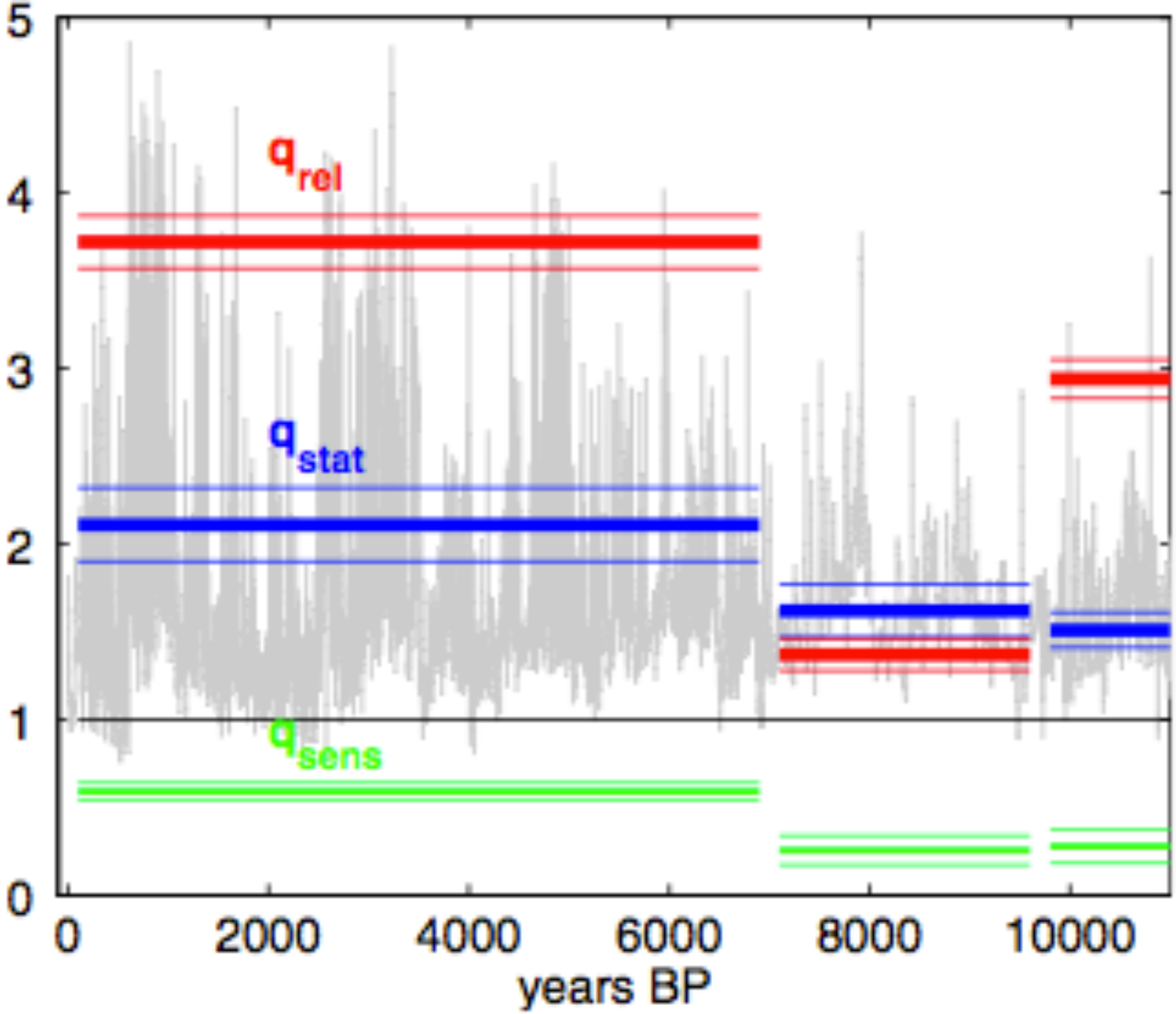

5
4
3
2
1
0
0
2000
4000
6000
8000
10000
years BP
q_{rel}
q_{stat}
q_{sens}

Physica A 391 (2012) 3069–3080

Contents lists available at SciVerse ScienceDirect

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journal homepage: www.elsevier.com/locate/physa

Tsallis statistics and magnetospheric self-organization

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Low dimensional chaos
Magnetosphere
Superstorm

ABSTRACT

In this study we use Tsallis non-extensive statistics for a new understanding the magnetospheric dynamics and the magnetospheric self-organization during quiet and intensive superstorm periods. The q_{sens}, q_{stat}, and q_{rel} indices set known as the Tsallis q-triplet was estimated during both quiet and strongly active periods, as well as the correlation dimensions and Lyapunov exponents spectrum for magnetospheric bulk plasma flows data. The results obtained by our analysis clearly indicate the magnetospheric phase transition process from a high-dimensional quiet SOC state to a low-dimensional global chaotic state when superstorm events are developed. During such a phase transition process the non-extensive statistical character of the magnetospheric plasma is strengthened as the values of the q-triplet indices changes obtaining higher values than their values during the quiet periods.

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Table 1

Summarize parameter values of magnetospheric dynamics: From the top to the bottom we show: changes of the ranges $\Delta\alpha$, $\Delta(D_q)$ of the multifractal profile. The *q*-triplet (*q*_sen, *q*_stat, *q*_rel) of Tsallis. The values of the maximum Lyapunov exponent (Li), the next Lyapunov exponent and the correlation dimension (*D*).

	Vx quiet	*Vx* storm
$\Delta\alpha = \alpha_{max} - \alpha_{min}$	1.069 ± 0.011	1.644 ± 0.03
$\Delta(D_q)$	0.721	1.205
*q*_sen	0.1343 ± 0.0267	0.3237 ± 0.0608
*q*_stat	1.120 ± 0.092	2.370 ± 0.056
*q*_rel	1.150 ± 0.080	2.910 ± 0.080
*L*1	≈ 0	>0
Li, $(i > 2)$	<0	<0
D (cor. Dim.)	>8	<4–5

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Nonextensivity in the solar magnetic activity during the increasing phase of solar cycle 23

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